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GROUP THEORY

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Lecture No. 2

Binary operations

- **Definition:** Let S be a nonempty set, a binary operation * is a function from the Cartesian product $\mathbf{S \times S}$ **into S**.
- **Mathematically**: Let $S \neq \emptyset$ and $*: S \times S \to S$ is a function s.t. $*(a, b) = a * b, \forall a, b \in S$.

Example1: Let $+: N \times N \to N$ is a function that is a binary operation on N since $\forall a, b \in N: +(a, b) = a + b \in N$.

Example2: $.: R \times R \to R$ is a binary operation on R.

Example3: $\div: Z \times Z \to Z$ is not binary operation since $\forall a, b \in Z: \div(a, b) = a \div b \notin Z$.

Definition: a mathematical system (mathematical structure), is a nonempty set of elements with one or more binary operations defined on this set.

Example: (N,+,.) is a math. sys., (R,.,÷) is a math. sys. (Z,.,÷) is not math. sys.,

Question1: let S={1,-1,i,-i} s.t. $i^2=-1$. Is (S,.) construct a math. system where (.) is an ordinary multiplication?

Question2: If Z_e and Z_O denote the even and odd integers respectively, are $(Z_e,+,.)$ &$(Z_o,+,.)$ constitute mathematical system?

Definition: The operation * defined on the set S is said to be associative if, $a * (b * c) = (a * b) * c : \; a, b, c \in S$.

Example1: + is an associative operation on N,Z,Q and R. also (.). but (-) is not asso. operation on R.

Example2: Let * be an operation defined on Z s.t. $\mathrm{a} * \mathrm{b} = \mathrm{a} + \mathrm{b} + \mathrm{ab}$: $\mathrm{a}, \mathrm{b} \in$ Z. Show whether that * is an associative operation on Z.

Sol. Let $a, b, c \in Z$

$$(a * b) * c = a * (b * c)$$

L.S. $(a * b) * c = (a + b + ab) * c$

$$= (a + b + ab) + c + (a + b + ab)c$$
$$= a + b + ab + c + ac + bc + abc$$

R.S. $a * (b * c) = a * (b + c + bc)$

$$= a + (b + c + bc) + a(b + c + bc)$$
$$= a + b + c + bc + ab + ac + abc$$

∵ L.S = R.S.

∴ * is an ass. operation on Z