University of Al-Hamdaniya, College of Education
Department of Mathematics

GROUP THEORY

Level Two

Asst. Lecturer. Hadil Hazim Sami

Lecture No. 6

Corollary: If (G,*) is a group then $(a^{-1})^{-1} = a \ \forall\, a \in G.$

Proof: Let $a \in G,$ since (G,*) is a group then $\exists\ a^{-1} \in G\ s.t.$

$$a * a^{-1} = a^{-1} * a = e \ldots (1)$$

Now, since $a^{-1} \in G, then\ \exists\ (a^{-1})^{-1} \in G\ s.t.$

$$a^{-1} * (a^{-1})^{-1} = (a^{-1})^{-1} * a^{-1} = e \ldots (2)$$

From (1) and (2) we have

$$a * a^{-1} = a^{-1} * (a^{-1})^{-1}$$

$\rightarrow a^{-1}$ has two inverse elements a and $(a^{-1})^{-1}$

But from (theorem 2) each element of a group (G,*) has exactly one inverse element.

$\therefore \ (a^{-1})^{-1} = a$.

Lemma: If $a, b, c, d \in G$ and (G,*) is a semi group then (a*b)*(c*d)=a*((b*c)*d)

Proof: L.S.) (a*b)*(c*d)

Let m=c*d

⇒(a*b)*(c*d)= (a*b)*m =a*(b*m) [since * associative]

= a*((b*(c*d))= a*((b*c)*d).

R.S.) $a*((b*c)*d) = a*((b*(c*d))$ [since * associative]

Let $m=c*d$

$\Rightarrow \quad a*(b*m) = (a*b)*m$ [since * associative]

$= (a*b)*(c*d)$

$\therefore (a*b)*(c*d)=a*((b*c)*d)$.

Theorem3: If (G,*) is a group, then $(a * b)^{-1} = b^{-1} * a^{-1}$ $\forall a, b \in G$.

Proof: ⇒ clearly $(a * b) * (a * b)^{-1} = (a * b)^{-1} * (a * b) = e$

⇐ $(a * b) * (b^{-1} * a^{-1}) = \left((a * b) * b^{-1}\right) * a^{-1}$ [*from the lemma*]

$= (a * (b * b^{-1})) * a^{-1} = (a * e) * a^{-1} = a * a^{-1} = e$

$\therefore \quad (a * b)^{-1} = b^{-1} * a^{-1}$

Corollary: If (G,*) is a commutative group, then $(a * b)^{-1}$

$= a^{-1} * b^{-1} \ \forall a, b \in G$.

Proof: Since (G,*) is a group then $(a * b)^{-1} = b^{-1} * a^{-1}$

[by th.3]

But $b^{-1} * a^{-1} = a^{-1} * b^{-1}$ [* comm. operation]

$\therefore (a * b)^{-1} = a^{-1} * b^{-1}$

H.W. Let G denotes the set of all ordered pairs of real numbers.

If the binary operation * is defined on the set G by the rule

$(a, b) * (c, d) = (ac, bc + d)$ then show that (G,*) is not commutative group? And find

$$((1,3) * (2,4))^{-1} \quad , (2,4)^{-1} * (1,3)^{-1}$$