University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 11

Problems

Q1/ If $(I_1, +, .)$ and $(I_2, +, .)$ are ideals of the ring (R,+,.) such that $I_1 \cap I_2 = \{0\}$ prove a.b=0 for every $a \in I_1$ and $b \in I_2$.

Proof: let $a \in I_1$ and $b \in I_2$

Since I_1 is an ideal, therefore a.b $\in I_1$

and since I_2 is an ideal therefore a.b$\in I_2$

$\therefore a.b \in I_1 \cap I_2 = \{0\}$

$\therefore a.b = 0$ for every $a \in I_1$ and $b \in I_2$.

Q2/Verify that the ring of real numbers (R,+,.) is a simple ring.

Proof: let (I,+,.) be a proper ideal of the ring (R,+,.) and let $0 \neq a \in I$

Since (R,+,.) has a multiplicative inverse therefore $\exists a^{-1} \in R$

$\rightarrow a.a^{-1} = 1 \in I.$

$\therefore R \subseteq I$ and since $I \subseteq R$

Therefore I=R C!

$\therefore (R,+,.)$is a simple ring.

Definition: a ring which contains no ideals except the trivial ideals is said to be a simple.

Q3/ Let (I,+,.) be an ideal of the ring$(R,+,.)$ and define $ann = \{r \in R: r.a = 0 \text{ for all } a \in I\}$.

prove that the triple (ann(I),+,.) constitutes an ideal of $(R,+,.)$ called the annihilator ideal of I.

Proof: since $0.a = 0$ for all $a \in I$

$\therefore ann(I) \neq \emptyset$ because $0 \in ann(I)$.

1) Let r_1 and $r_2 \in ann(I)$. that is $r_1.a = 0 \; \& \; r_2.a = 0 \;\; \forall a \in I$

$(r_1 - r_2).a = r_1a - r_2a = 0 - 0 = 0$

$\therefore r_1 - r_2 \in ann(I)$

2) Let $x \in ann(I)$ and $r \in R$

(xr)a=x(ra)=0

$\therefore xr \in ann(I)$

Definition: Let (R ,+,.) be a ring and ∅≠I⊆R then (I,+,.) is an ideal of (R ,+,.) iff:

1. a-b∈I ∀ a,b ∈I
2. ar∈I and ra∈I ∀r∈R,a∈I

Similarly

$(rx)a = r(xa) = r.0 = 0$

$\therefore\ rx \in \ ann(I)$

$\therefore$ (ann(I),+,.) is an ideal of $(R, +, .)$.

Example: let (Z_{10},+,.) is a ring , (2)&(5) are proper ideal of Z_{10}. find ann(2) and ann(5)

Sol..;

0.0=0 , 0.2=0,…

1.0=0, 1.2=2 $\neq$ 0

2.0=0, 2.2=4 $\neq$ 0

⋮

5.0=0, 5.2=0, 5.4=0, 5.6=0, 5.8=0

⋮

9.0=0, 9.2=18=8 $\neq$ 0

ann(2)={0,5},

ann(5)=? H.W