



University of Al-Hamdaniya, College of  
Education

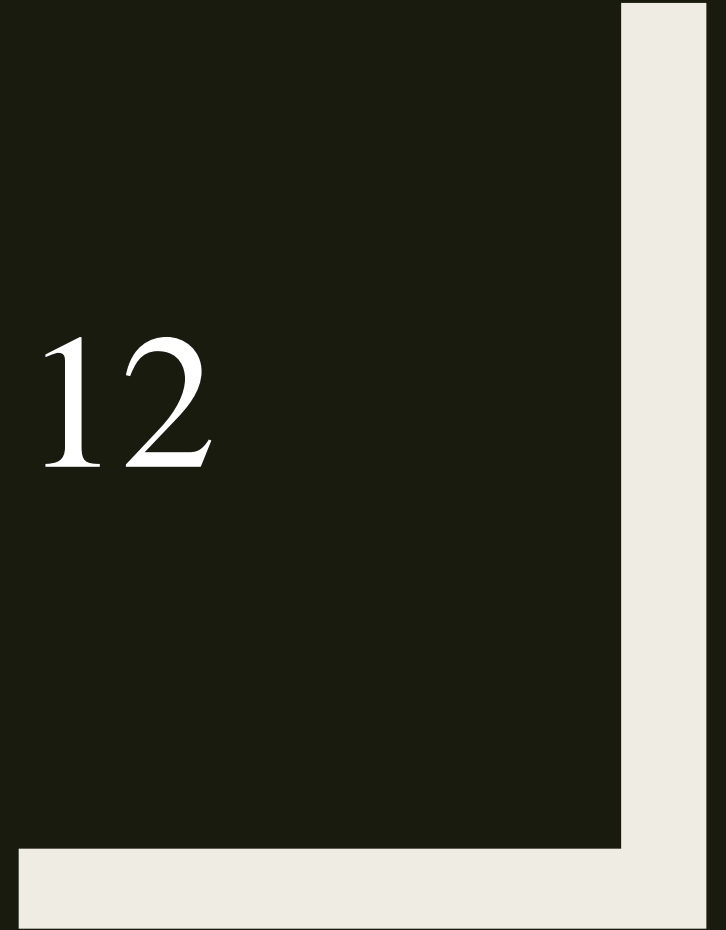
Department of Mathematics

RING THEORY

Level Three

Asst. Lecturer. Hadil Hazim Sami

# LECTURE NO. 12



# Problems

Q4/ suppose  $(I_1, +, \cdot)$  and  $(I_2, +, \cdot)$  are ideals of the ring  $(R, +, \cdot)$ . Define  $I_1 + I_2 = \{a + b : a$

Proof: since  $0 \in I_1$  &  $0 \in I_2$ , hence  $0=0+0$

$$\therefore I_1 + I_2 \neq \emptyset$$

1) Let  $a_1 + b_1$  and  $a_2 + b_2 \in I_1 + I_2$

$$(a_1 + b_1) - (a_2 + b_2) = (a_1 - a_2) + (b_1 - b_2) \in I_1 + I_2$$

**Definition:** Let  $(R, +, \cdot)$  be a ring and  $\emptyset \neq I \subseteq R$  then  $(I, +, \cdot)$  is an ideal of  $(R, +, \cdot)$  iff:

1.  $a - b \in I \quad \forall a, b \in I$
2.  $ar \in I$  and  $ra \in I$   
 $\forall r \in R, a \in I$

2) Let  $a + b \in I_1 + I_2$  and  $r \in R$

$$(a+b)r=ar+br \in I_1 + I_2$$

Similarly

$$r(a+b)=ra+rb$$

$\therefore (I_1 + I_2, +, \cdot)$  is an ideal of the ring  $(R, +, \cdot)$ .

Example: if  $(\mathbb{Z}_6, +, \cdot)$  is a ring,  $(2)$  &  $(3)$  are proper ideal of  $\mathbb{Z}_6$ , then

$$(2) + (3) = \{0, 2, 4\} + \{0, 3\}$$

$$= \{0, 1, 2, 3, 4, 5\} = \mathbb{Z}_6$$

**Definition** : Let  $R$  be a commutative ring with identity and an ideal  $I=(a)$  generated by an element  $a \in R$  is called a principal ideal of the ring  $(R,+,\cdot)$  and defined by

$$(a) = aR = \{ar : \forall r \in R, a \in R\} .$$

Prove  $(a) = \{ar : \forall r \in R, a \in R\}$  is an ideal.(H.W)

Example: consider the ring  $(\mathbb{Z},+,\cdot)$

$$(2) = \{(2)r : \forall r \in \mathbb{Z}\} = \{0, \mp 2, \mp 4, \dots\}$$

$$(-2) = \{(-2)r : \forall r \in \mathbb{Z}\} = \{0, \pm 2, \pm 4, \dots\}$$

$$\rightarrow (2) = (-2)$$

$$(1) = \mathbb{Z}$$

$$(0) = \{0\}$$