University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 12

Problems

Q4/ suppose $(I_1,+,.)$ and $(I_2,+,.)$ are ideals of the ring $(R,+,.)$. Define $I_1 + I_2 = \{a + b: a$

Proof: since $0 \in I_1$ & $0 \in I_2$, hence 0=0+0

$$\therefore I_1 + I_2 \neq \emptyset$$

1) Let $a_1 + b_1$ and $a_2 + b_2 \in I_1 + I_2$

$$(a_1 + b_1) - (a_2 + b_2) = (a_1 - a_2) + (b_1 - b_2) \in I_1 + I_2$$

Definition: Let (R ,+,.) be a ring and ∅≠I⊆R then (I,+,.) is an ideal of (R ,+,.) iff:

1. a-b∈I ∀ a,b ∈I
2. ar∈I and ra∈I ∀r∈R,a∈I

2) Let $a + b \in I_1 + I_2$ and $r \in R$

$(a+b)r = ar + br \in I_1 + I_2$

Similarly

$r(a+b) = ra + rb$

$\therefore$ $(I_1 + I_2, +, .)$ is an ideal of the ring $(R, +, .)$.

<u>Example: if</u> $(Z_6,+,.)$ is a ring , (2)&(3) are proper ideal of $Z_{6\,,}$ then

$$(2)+(3)=\{0,2,4\}+\{0,3\}$$

$$=\{0,1,2,3,4,5\}= Z_6$$

Definition : Let R be a commutative ring with identity and an ideal I=(a) generated by an element $a \in R$ is called a principal ideal of the ring (R,+,.) and defined by

$$(a) = aR = \{ar: \forall r \in R, a \in R\}.$$

Prove $(a) = \{ar: \forall r \in R, a \in R\}$ is an ideal.(H.W)

Example: consider the ring (Z,+,.)

$$(2) = \{(2)r: \quad \forall r \in Z\} = \{0, \mp 2, \mp 4, \dots\}$$

$$(-2) = \{(-2)r: \quad \forall r \in Z\} = \{0, \pm 2, \pm 4, \dots\}$$

$$\rightarrow (2) = (-2)$$

$$(1) = Z$$

$$(0) = \{0\}$$