University of Al-Hamdaniya, College of Education
Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 13

Definition: Let R be a commutative ring with identity and $(I_i,+,.)$ are ideals of R where i=0,1,2,…,n then (R,+,.) is called a principle ideal ring (P.I.R) if and only if every ideal of R is a principal ideal.

Definition : Let R be a commutative ring with identity and an ideal I=(a) generated by an element a ∈ R is called a principal ideal of the ring (R,+,.) and defined by

$(a) = aR = \{ar : \forall r \in R, a \in R\}$.

Example: (Z,+,.) is a principle ideal ring, since (Z,+,.) is a commutative ring with identity in which every ideals of (Z,+,.)are of the form ((n),+,.) where n is a nonnegative integer.

Theorem8: Let I_1 and I_2 be ideals of the ring $(R,+,.)$. A ring R is said to be direct sum of I_1 and I_2 if :

1) $I_1 + I_2 = R$

2) $I_1 \cap I_2 = \{0\}$

and denoted by $R = I_1 \oplus I_2$.

Example: $(Z_{12},+,.)$ is a ring and $I_1 = \{0,3,6,9\}$ and $I_2 = \{0,4,8\}$ are ideals of Z_{12}, show that $(Z_{12},+,.)$ is a direct sum of I_1 and I_2.

Solution: 1) $I_1 + I_2 = \{0,3,6,9\} + \{0,4,8\} = Z_{12}$

2) $I_1 \cap I_2 = \{0\}$

$\therefore Z_{12}$ is a direct sum of I_1 and I_2.

Definition: Let (R,+,.) be a ring and $a \in R$ then a is said to be an idempotent element if $a^2 = a$.

Example: $(Z_6,+,.)$ is a ring find all the idempotent element of Z_6.

Solution: $0^2 = 0$,

$1^2 = 1$

$2^2 = 4 \neq 2$,

$3^2 = 9 = 3$

$4^2 = 16 = 4$

$5^2 = 25 = 1 \neq 5$.

$\therefore$ Idempotent element of Z_6 are $\{0,1,3,4\}$