

University of Al-Hamdaniya, College of  
Education

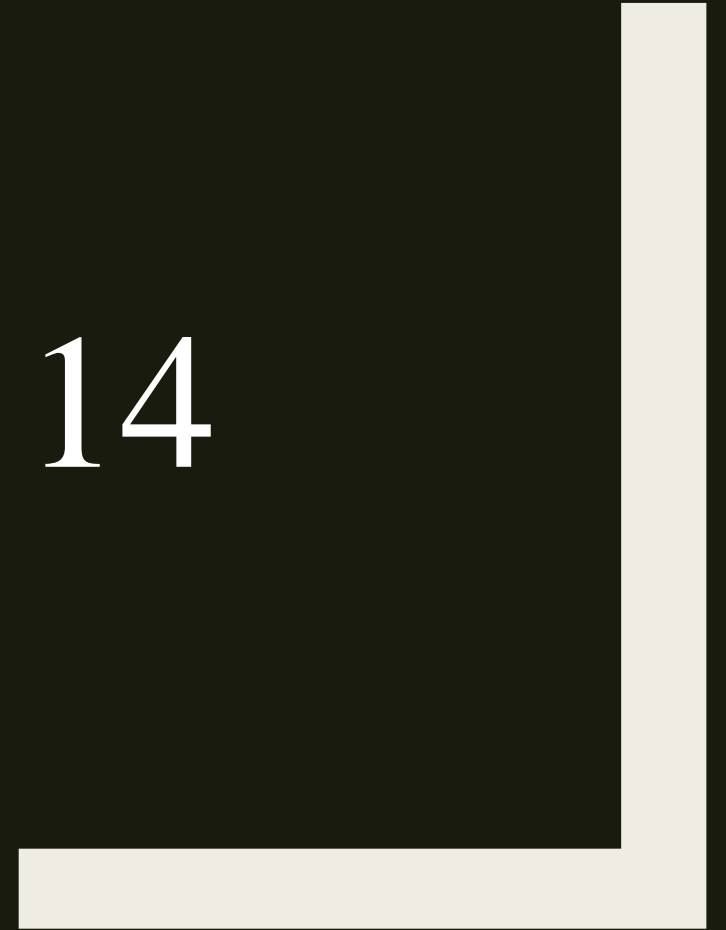
Department of Mathematics

RING THEORY

Level Three

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# LECTURE NO. 14



**Definition:** Let  $(R, +, \cdot)$  and  $(R', +', \cdot')$  are rings and let  $f: R \rightarrow R'$  be a function then  $f$  is said to be a ring homomorphism iff:

$$1) f(a+b) = f(a) +' f(b)$$

$$2) f(a \cdot b) = f(a) \cdot' f(b)$$

$$\forall a, b \in R.$$

**Example(1):** Let  $f: (\mathbb{Z}, +, \cdot) \rightarrow (\mathbb{Z}, +, \cdot)$  is a function defined by:

$$f(a) = 0 \quad \forall a \in \mathbb{Z}.$$

Is  $f$  homo. ring?

sol: Let  $a, b \in \mathbb{Z}$

$$1) f(a+b) = 0$$

$$= 0+0$$

$$= f(a) + f(b)$$

$$2) f(a \cdot b) = 0$$

$$= 0 \cdot 0$$

$$= f(a) \cdot f(b)$$



$\therefore f$  is ring homomorphism.

Example(2): Let  $f: (R, +, \cdot) \rightarrow (R, +, \cdot)$  is a function defined by:

$$f(a) = 2a \quad \forall a \in R \text{ is } f \text{ homo. ring?}$$

Sol: Let  $a, b \in R$

$$1) f(a+b) = 2(a+b)$$

$$= 2a + 2b$$

$$= f(a) + f(b)$$

$$2) f(a \cdot b) = 2(ab)$$

$$\neq f(a) \cdot f(b)$$

$\therefore f$  is not ring homomorphism.