University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

Asst. Lecturer. Hadil Hazim Sami

LECTURE NO. 16

Theorem8: Let f be a ring homomorphism from (R, + ,.)into the ring (R′,+′,.′) then f is one to one iff $\ker(f) = \{0\}$.

Proof: Let $\ker(f) = \{0\}$ and let f(a)=f(b) ; $a, b \in R$

$\Rightarrow f(a) - f(b) = 0'$

$\Rightarrow f(a - b) = 0'$[since f is homo.]

$\Rightarrow a - b \in \ker(f) = \{0\}$

$\Rightarrow a - b \in \ker(f) = \{0\}$

$\Rightarrow a - b = 0$

توضيح
شرط التباين

f(a)=f(b) ⇒ a=b

العنصر عندما ينتمي الى kernel

If $b \in \ker(f)$ ⇒ f(b)=0’

$\Rightarrow a = b$

$\therefore$ f is one to one

<u>Conversely</u>: Let f be $1-1$ and let $a \in \ker(f) \Rightarrow f(a) = 0'$

but $f\left(0\right) = 0'$

$\Rightarrow f(a) = f(0)$

and since f is $1-1$

$\Rightarrow a = 0$

$\therefore \ker(f) = \{0\}$