



University of Al-Hamdaniya, College of
Education

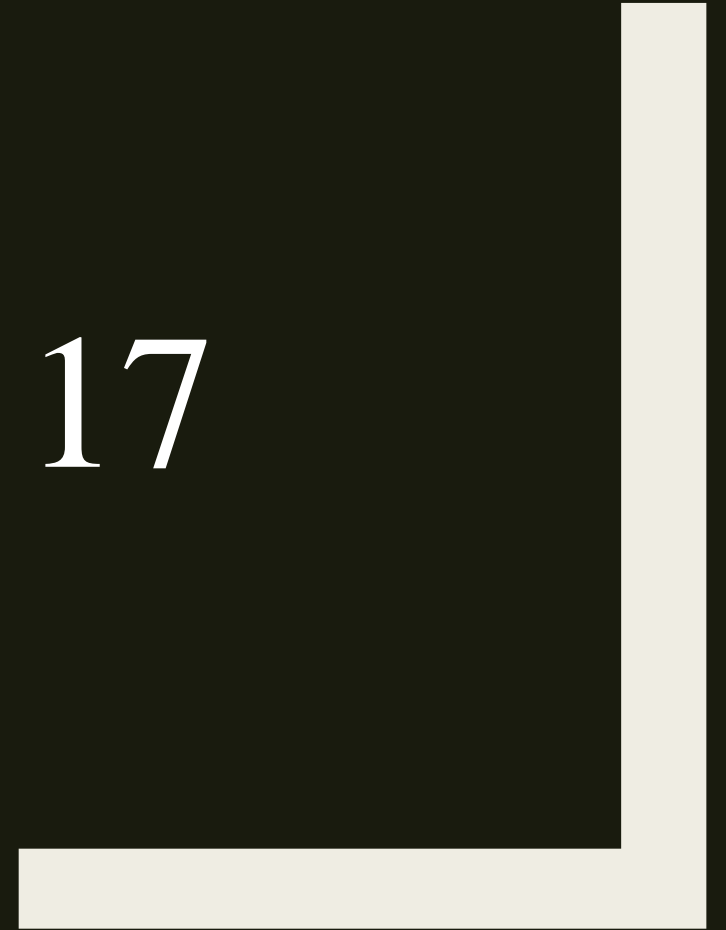
Department of Mathematics

RING THEORY

Level Three

Asst. Lecturer. Hadil Hazim Sami

LECTURE NO. 17



Theorem9: Let f be a ring homomorphism from $(R, +, \cdot)$ into the ring $(R', +', \cdot')$ then $(\ker(f), +, \cdot)$ is a subring of the ring $(R, +, \cdot)$.

Proof: 1) since $f(0) = 0'$

$$\Rightarrow 0 \in \ker(f)$$

$$\Rightarrow \ker(f) \neq \emptyset$$

2) Let $x, y \in \ker(f)$

$$\Rightarrow f(x) = 0' \quad , f(y) = 0'$$

$$\Rightarrow f(x) +' (-f(y)) = 0' +' 0' = 0'$$

Definition2: Let $(R, +, \cdot)$ be a ring and let $\emptyset \neq S \subseteq R$ then $(S, +, \cdot)$ is a subring of $(R, +, \cdot)$ iff:

$$1) a - b \in S \quad \forall a, b \in S.$$

$$2) a \cdot b \in S \quad \forall a, b \in S.$$

العنصر عندما ينتمي الى
kernel

If $b \in \ker(f)$

$f(b) = 0'$

$$= f(x + (-y))$$

$$\Leftrightarrow x + (-y) \in \ker(f)$$

$$3) f(x) \cdot f(y) = 0' \cdot 0' = 0'$$

$$= f(x \cdot y)$$

$$\Leftrightarrow x \cdot y \in \ker(f)$$

$\therefore \ker(f)$ is subring of $(R, +, \cdot)$.

Theorem10: Let f be a ring homomorphism from $(R, +, \cdot)$ into the ring $(R', +', \cdot')$ then $(\ker(f), +, \cdot)$ is an ideal of the ring $(R, +, \cdot)$.

Proof: 1) $\ker(f) \neq \emptyset$ since $0 \in \ker(f)$

2) Let $a, b \in \ker(f)$

$$\Rightarrow f(a) = 0' \quad , f(b) = 0'$$

$$f(a - b) = f(a + (-b))$$

$$= f(a) +' f(-b)$$

$$= 0' +' 0' = 0'$$

$$\Rightarrow a - b \in \ker(f)$$

Definition: Let $(R, +, \cdot)$ be a ring and $\emptyset \neq I \subseteq R$ then $(I, +, \cdot)$ is an ideal of $(R, +, \cdot)$ iff:

1. $a - b \in I \quad \forall a, b \in I$
2. $ar \in I$ and $ra \in I \quad \forall r \in R, a \in I$

3) Let $a \in \ker(f)$, $r \in R$

$$\Rightarrow f(a) = 0'$$

$$f(ar) = f(a) \cdot f(r) \quad [\text{since } f \text{ is homo.}]$$

$$= 0' \cdot f(r)$$

$$= 0'$$

$$\therefore ar \in \ker(f)$$

Similarly $f(ra) = 0'$

$$\therefore ra \in \ker(f)$$

$\therefore \ker(f)$ is an ideal of the ring $(R, +, \cdot)$.