University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 17

Theorem9: Let f be a ring homomorphism from $(R, + ,.)$ into the ring $(R',+',.')$ then $(\ker(f), + ,.)$ is a subring of the ring $(R, + ,.)$.

Proof: 1) since $f(0)=0'$

$\Rightarrow 0 \in \ker(f)$

$\Rightarrow \ker(f) \neq \emptyset$

2) Let $x, y \in \ker(f)$

$\Rightarrow f(x) = 0' \quad , f(y) = 0'$

$\Rightarrow f(x) +' \left(-f(y)\right) = 0' +' 0' = 0'$

Definition2: Let $(R,+,.)$ be a ring and let $\emptyset \neq S \subseteq R$ then $(S,+,.)$ is a subring of $(R,+,.)$ iff:

1) $a - b \in S \;\; \forall \;\; a, b \in S.$

2) $a.b \in S \;\; \forall \;\; a, b \in S.$

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kernel

If $b \in \ker(f)$ $\quad$ f(b)=0'

$= f(x + (-y))$

$\Rightarrow x + (-y) \in \ker(f)$

3) $f(x).' \ f(y) = 0'.' \, 0' = 0'$

$= f(x.y)$

$\Rightarrow x.y \in \ker(f)$

$\therefore$ $\ker(f)$ is subring of $(R, +, .)$.

Theorem10: Let f be a ring homomorphism from $(R,+,.)$ into the ring $(R',+',.')$ then $(\ker(f),+,.)$ is an ideal of the ring $(R,+,.)$.

Proof: 1) $\ker(f) \neq \emptyset$ since $0 \in \ker(f)$

2) Let $a, b \in \ker(f)$

$\Rightarrow f(a) = 0' \quad , f(b) = 0'$

$f(a-b) = f\big(a+(-b)\big)$

$= f(a) +' f(-b)$

$= 0' +' 0' = 0'$

$\Rightarrow a - b \in \ker(f)$

Definition: Let (R ,+,.) be a ring and ∅≠I⊆R then (I,+,.) is an ideal of (R ,+,.) iff:

1. a-b∈I ∀ a,b ∈I
2. ar∈I and ra∈I ∀r∈R,a∈I

3) Let $a \in \ker(f)$, $r \in R$

$\Rightarrow f(a) = 0'$

$f(ar) = f(a).'f(r)$ [since f is homo.]

$= 0'.'f(r)$

$= 0'$

$\therefore ar \in \ker(f)$

Similarly $f(ra) = 0'$

$\therefore ra \in \ker(f)$

$\therefore \ker(f)$ is an ideal of the ring $(R, +, .)$.