University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 18

Theorem11: Let f be a ring homomorphism from $(R, +, .)$ into the ring $(R', +', .')$ then:

1) $f(0)=0'$

Proof: 1) $f(0) = f(0+0) = f(0) +' f(0)$ [since f is homo.]

$f(0) +' 0' = f(0) +' f(0)$

$f(0) = 0'$

2) $f(-a) = -f(a) \quad a \in R.$

proof2): $a + (-a) = 0$

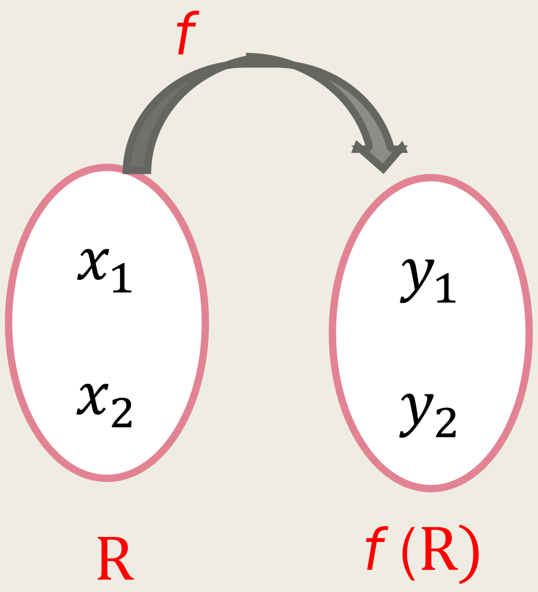
$\Rightarrow f\big(a + (-a)\big) = f(0)$

Since f is homo. & from (1) we have:

$f(a) +' f(-a) = 0'$

$f(-a) = -f(a) \quad \forall a \in R$

3) $(f(R),+',.')$ is a subring of $(R',+',.')$.



Proof:3) $\because 0 \in R \rightarrow f(0) = 0'$

$\Rightarrow f(R) \neq \emptyset$

$\forall y_1, y_2 \in f(R) \; \exists \; x_1, x_2 \in R \; s.t.$

$f(x_1) = y_1 \; \& \; f(x_2) = y_2$

$y_1 +' (-y_2) = f(x_1) +' (-f(x_2)) = f(x_1 - x_2) \in f(R)$ $[since\ f\ is\ homo.]$

$$y_1 .' \; y_2 = f(x_1) .' f(x_2) = f(x_1 . x_2) \in f(R) \; [since \; f \; is \; homo.]$$

$\therefore f(R)$ is subring of $(R', +', .')$