



University of Al-Hamdaniya, College of
Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 2



Definition: let $(S, *, \circ)$ be a mathematical system with binary operations $*$ and \circ . The operation (\circ) is said to be distributive over the operation $(*)$ if :

$$1) a \circ (b * c) = (a \circ b) * (a \circ c) \quad [\text{left dist. Law}]$$

$$2) (b * c) \circ a = (b \circ a) * (c \circ a) \quad [\text{right dist. Law}]$$

$$\forall a, b, c \in S.$$

Example (1): the binary operation $(.)$ is distributive over the binary operation $(+)$ on a set R , since:

$$1) a \cdot (b + c) = (a \cdot b) + (a \cdot c) \quad [\text{left dist. Law}]$$

$$2) (b + c) \cdot a = (b \cdot a) + (c \cdot a) \quad [\text{right dist. Law}]$$

$$\forall a, b, c \in R.$$

Example (2): The two binary operation $*$ & \circ defined on the set of integers (\mathbb{Z}) by :

$$a * b = a + 2b$$

$$a \circ b = 2ab$$

$$\forall a, b \in \mathbb{Z}.$$

Is \circ distributive over the binary operation $*$?

Solution: 1) $a \circ (b * c) = (a \circ b) * (a \circ c)$

$$\begin{aligned} \text{L.S) } a \circ (b * c) &= a \circ (b + 2c) \\ &= 2ab + 4ac \end{aligned}$$

$$\begin{aligned} \text{R.S) } (a \circ b) * (a \circ c) &= (2ab) * (2ac) \\ &= 2ab + 4ac \end{aligned}$$

since L. S = R. S

$$\therefore a \circ (b * c) = (a \circ b) * (a \circ c)$$

$$2) (b * c) \circ a = (b \circ a) * (c \circ a)$$

$$\text{L.S) } (b * c) \circ a = (b+2c) \circ a$$

$$= 2ba + 4ca$$

$$\text{R.S) } (b \circ a) * (c \circ a) = (2ba) * (2ca)$$

$$= 2ba + 4ca$$

since L.S=R.S

$$\therefore (b * c) \circ a = (b \circ a) * (c \circ a)$$

From (1) & (2) we show that \circ is distributive over $*$.

Note: If (\circ) is a commutative operation and satisfies the left or right distributive law, we conclude that (\circ) is distributive over $(*)$.