



University of Al-Hamdaniya, College of Education Department of Mathematics RING THEORY Level Three Asst. Lecturer. Hadil Hazim Sami

LECTURE NO. 20

Ring Isomorphism

<u>Definition</u> : If (R,+,.) and (R',+',.') are two rings, let f be a function from R into R' i.e.

f: $R \rightarrow R'$, then f is called a ring isomorphism if:

1) f is a ring homomorphism.

1) f(a+b)=f(a)+' f(b)2) f(a.b)=f(a).' f(b)

2)f is one to one (injective)

3) f is onto (surjective)

$$\forall y \in R' \exists x \in R \exists f(x) = y$$

 $f(a)=f(b) \implies a=b \forall a, b \in R$

Question/ Let R and *R* are rings and Δ_1, Δ_2 are two binary operations of R s.t.

 $x \Delta_1 y = x + y + 1$

 $x \Delta_2 y = x + y + xy \quad \forall x, y \in R$

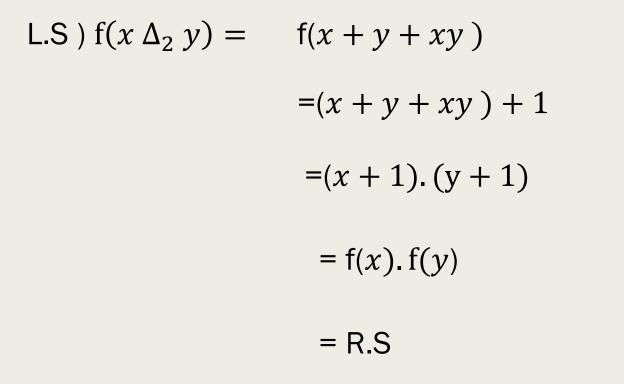
Show that f: $(R, \Delta_1, \Delta_2) \rightarrow (R, +, .)$ is a ring isomorphism s.t

 $f(x) = x + 1 \ \forall x \in R$

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Solution: 1) Let x, y \in \mathbb{R} then:
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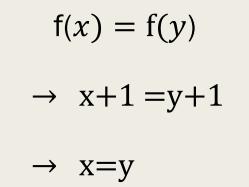
L.S)
$$f(x \Delta_1 y) = f(x + y + 1)$$

= $(x + y + 1) + 1$
= $(x + 1) + (y + 1)$
= $(x + 1) + (y + 1)$
= $f(x) + f(y)$
= R.S



 \therefore f is homo.

2) Let $x, y \in \mathbb{R}$ s.t:



\therefore f is 1-1

3) $\forall y \in R \exists y - 1 \in R \ni f(y - 1) = y$

- \therefore f is onto
- \therefore f is a ring isomorphism

<u>**Theorem 12:**</u> natural mapping (nat_I) is a homo. From the ring (R,+,.) into the ring $(R/I, \bigoplus, \bigotimes)$ with ker (natI) = I.

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Proof: let nat<sub>I</sub>: R \rightarrow R/I defined by :
nat<sub>I</sub>(a) = a + I , a \in R
1) let a + I, b + I \in R/I \quad \forall a, b \in R
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L.S.) nat_{I}(a + b) = (a + b) + I
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= (a + I) \oplus (b + I)
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 $= nat_I(a) \oplus nat_I(b)$

= R.S.

2)
$$nat_{I}(ab) = (ab) + 1$$

$$= (a + I) \otimes (b + I)$$

$$= \operatorname{nat}_{I}(a) \otimes \operatorname{nat}_{I}(b)$$

 \therefore nat₁ is a homo.

 $ker(nat_{I}) = \{a \in R: nat_{I}(a) = 0 + I\}$

$$= \{a \in R: a + I = 0 + I\}$$

$$= \{a \in \mathbb{R} : a + I = I\}$$

 $= \{a \in R: a \in I\}$

= I

<u>**H.w**</u>: $(Z_4, +, .)$ ring, I={0,2}. Show that nat_I is a homo. from $Z_4 \rightarrow Z_4/I$. and find ker (nat_I).