University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 21

Problem

Q/ Given that f is a ring homo. From the ring (R,+,.) onto the ring $(R', +', .')$ prove that:

a) If $(I, +, .)$ is an ideal of $(R, +, .)$ then the triple $(f(I), +', .')$ is an ideal of $(R', +', .')$.

Proof: 1) since $0 \in I$, then $f(0) = o' \in f(I)$

$\therefore f(I) \neq \emptyset$

2)let $f(a), f(b) \in f(I)$ then $a, b \in I$ and since I is an ideal of R then $a - b \in I$

$\Rightarrow f(a) - f(b) = f(a - b) \in f(I)$ [f is homo.]

2)let $f(a) \in f(I)$, $f(r) \in R'$[f is onto]

$\rightarrow f(r).f(a) =$ $f(ra) \in f(I)$ [since $ra \in I$, I is an ideal]

$\therefore f(r).f(a) \in f(I)$

Similarly: $f(a).f(r) = f(ar) \in f(I)$ [since $ra \in I$, I is an ideal]

$\therefore f(a).f(r) \in f(I)$

$\therefore f(I)$ is an ideal of R'.

b) If $(I', +', .')$ is an ideal of $(R', +', .')$ then the triple $(f^{-1}(I'),+,.)$ is an ideal of $(R, +, .)$ with $\ker(f) \subseteq f^{-1}(I')$

Proof:1) since $o' \in I'$ and $f(0) = o'$ then $0 \in f^{-1}(I')$

$\therefore f^{-1}(I') \neq \emptyset$

2) Let $a, b \in f^{-1}(I')$ then $f(a), f(b) \in I'$

so $f(a - b) = f(a) - f(b) \in I'$

$\therefore$ a-b$\in f^{-1}(I')$

3) Let $a \in f^{-1}(I'), r \in R$ ⇨$f(a) \in I'$ & $f(r) \in R'$

$f(ar) = f(a).f(r) \in I'$ [I' is an ideal]

$\therefore ar \in f^{-1}(I')$

Similarly: $f(ra) = f(r).f(a) \in I'$ [I' is an ideal]

$\therefore ra \in f^{-1}(I')$

$\therefore f^{-1}(I')$ is an ideal of R

To prove that ker(f) $\subseteq \mathbf{f^{-1}(I')}$

Let $a \in \ker(f) \Rightarrow f(a) = o' \in I'$

$\Rightarrow \ a \in f^{-1}(0')$

$\Rightarrow a \in f^{-1}(I')$ [since $0' \in I'$]

$\therefore \ker(f) \subseteq f^{-1}(I')$.