

University of Al-Hamdaniya, College of  
Education

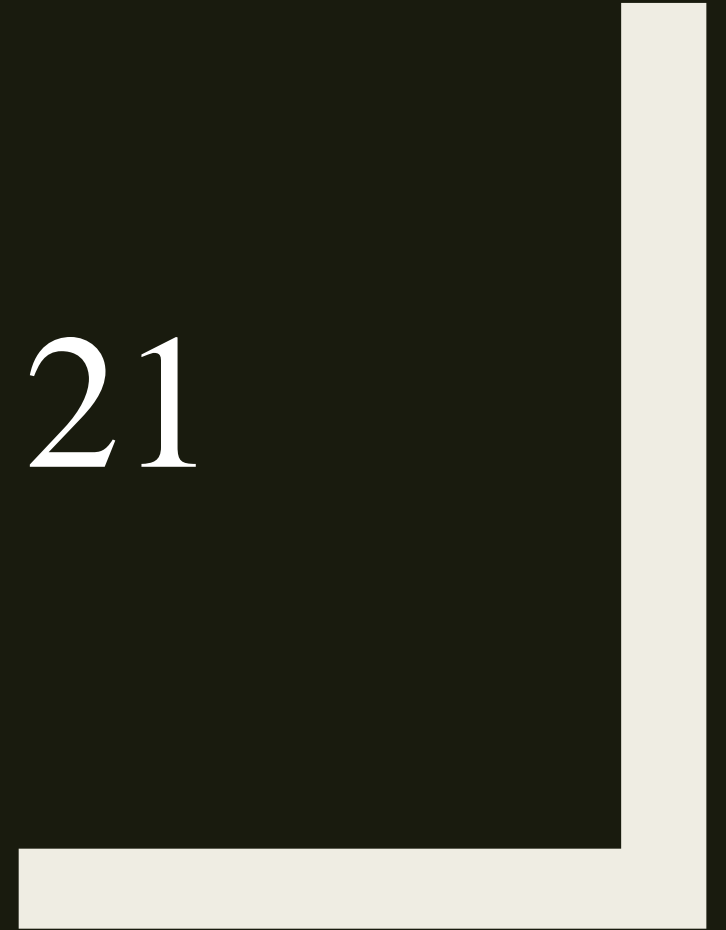
Department of Mathematics

RING THEORY

Level Three

Asst. Lecturer. Hadil Hazim Sami

# LECTURE NO. 21



## Problem

Q/ Given that  $f$  is a ring homo. From the ring  $(R, +, \cdot)$  onto the ring  $(R', +', \cdot')$  prove that:

a) If  $(I, +, \cdot)$  is an ideal of  $(R, +, \cdot)$  then the triple  $(f(I), +', \cdot')$  is an ideal of  $(R', +', \cdot')$  .

**Proof:** 1) since  $0 \in I$ , then  $f(0) = o' \in f(I)$

$\therefore f(I) \neq \emptyset$

2) let  $f(a), f(b) \in f(I)$  then  $a, b \in I$  and since  $I$  is an ideal of  $R$  then  $a - b \in I$

$\Rightarrow f(a) - f(b) = f(a - b) \in f(I)$  [f is homo.]

2) let  $f(a) \in f(I)$ ,  $f(r) \in R'$  [  $f$  is onto ]

$\rightarrow f(r) \cdot f(a) = f(ra) \in f(I)$  [since  $ra \in I$ ,  $I$  is an ideal]

$\therefore f(r) \cdot f(a) \in f(I)$

**Similarly:**  $f(a) \cdot f(r) = f(ar) \in f(I)$  [since  $ar \in I$ ,  $I$  is an ideal]

$\therefore f(a) \cdot f(r) \in f(I)$

$\therefore f(I)$  is an ideal of  $R'$ .

b) If  $(I', +', \cdot')$  is an ideal of  $(R', +', \cdot')$  then the triple  $(f^{-1}(I'), +, \cdot)$  is an ideal of  $(R, +, \cdot)$  with  $\ker(f) \subseteq f^{-1}(I')$

**Proof:**1) since  $o' \in I'$  and  $f(0) = o'$  then  $0 \in f^{-1}(I')$

$$\therefore f^{-1}(I') \neq \emptyset$$

2) Let  $a, b \in f^{-1}(I')$  then  $f(a), f(b) \in I'$

$$\text{so } f(a - b) = f(a) - f(b) \in I'$$

$$\therefore a - b \in f^{-1}(I')$$

3) Let  $a \in f^{-1}(I'), r \in R \Rightarrow f(a) \in I' \& f(r) \in R'$

$f(ar) = f(a).f(r) \in I' \quad [I' \text{ is an ideal}]$

$\therefore ar \in f^{-1}(I')$

**Similarly:**  $f(ra) = f(r).f(a) \in I' \quad [I' \text{ is an ideal}]$

$\therefore ra \in f^{-1}(I')$

$\therefore f^{-1}(I') \text{ is an ideal of } R$

To prove that  $\ker(f) \subseteq f^{-1}(I')$

Let  $a \in \ker(f) \Rightarrow f(a) = o' \in I'$

$\Rightarrow a \in f^{-1}(o')$

$\Rightarrow a \in f^{-1}(I')$  [since  $o' \in I'$ ]

$\therefore \ker(f) \subseteq f^{-1}(I')$ .