University of Al-Hamdaniya, College of Education
Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 5

Corollary: suppose that R be a ring with identity 1 and has no zero divisors then $a^2 = a \rightarrow a = 0$ or $a = 1$

Proof: $a^2 = a$

$$\rightarrow a^2 - a = 0$$

$$a(a-1) = 0 \text{ [since R has no zero divisors]}$$

$$\rightarrow a = 0 \text{ or } a - 1 = 0$$

$$\rightarrow a = 1$$

$\therefore a = 0$ or $a = 1$.

Definition: An integral domain is a comm. ring with identity without zero divisor.

Example1: (Z,+,.), (R,+,.), (Q,+,.), (C,+,.) are integral domain.

Example2: (Zn,+n,.n)

If n= prime ⇨ ID

If n= Not prime ⇨ not ID

H.W. Is (Ze,+e,.e) an integral domain?

Subring

Definition1: let (R,+,.) be a ring, let $\emptyset \neq S \subseteq R$ then S is called a subring of R iff (S,+,.) is a ring.

Definition2: Let (R,+,.) be a ring and let $\emptyset \neq S \subseteq R$ then (S,+,.) is a subring of (R,+,.) iff:

1) $a - b \in S \ \forall \ a, b \in S.$
2) $a.b \in S \ \forall \ a, b \in S.$

Note: Every ring (R,*, ₒ) has two trivial subring ({e},*,ₒ) and (R,*, ₒ).

Example: In the ring of integers (Z,+,.) the triple (Ze,+,.) is a subring of Z. while (Zo,+,.) is not subring of Z. a

Q/ Let $S = \{a + b\sqrt{3} : a, b \in Z\}$ show that (S,+,.) subring of (R,+,.).

Sol/ Let $a_1 + b_1\sqrt{3}$, $a_2 + b_2\sqrt{3} \in S$

1) $(a_1+b_1\sqrt{3}) - (a_2+b_2\sqrt{3}) = (a_1 - a_2) + (b_1 - b_2)\sqrt{3} \in S$

2) $(a_1+b_1\sqrt{3}) . (a_2+b_2\sqrt{3}) = a_1a_2 + a_1b_2\sqrt{3} + b_1\sqrt{3}\, a_2 + 3b_1b_2$

$\therefore S$ is a subring of (R,+,.).

<u>Q2</u>/ (Z6,+,.) is a ring , and subring of (Z6,+,.) are :

S1= {0}

S2={Z6}

S3= {0,2,4}

S4= {0,3}

Remark: Let R be a ring and S be a subring of R.

1) If R is a comm. then S is a comm.
2) If S is comm. then is not necessary that R is comm.
3) If R has identity, then it is not necessary that S has identity.
4) If may that R , S have the same identity .
5) If may be that R , S have distinct identity.