

University of Al-Hamdaniya, College of
Education

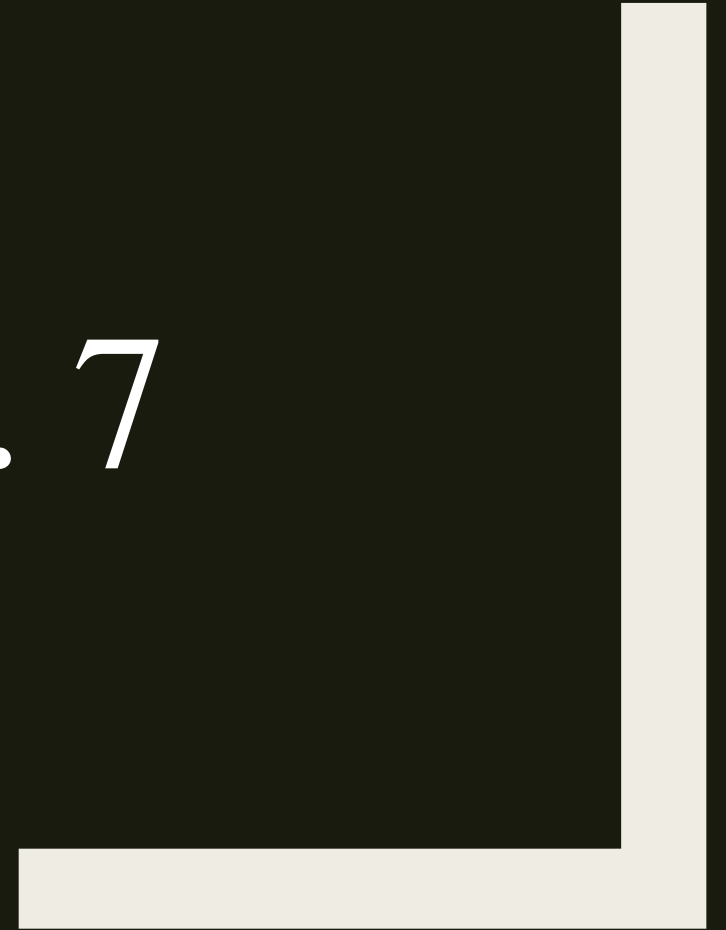
Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 7



Problems

Q1/ Define two binary operations $*$ and \circ on the set Z of integers as follows:

$$a * b = a + b - 1$$

$$a \circ b = a + b - ab$$

prove that the system $(Z, *, \circ)$ is comm. ring with identity?

Q2/ Let $(R,+)$ comm. group. Determine whether $(R,+.)$ forms a ring with multiplication defined as:

$$\forall a, b \in R$$

1) $a \cdot b = a$

sol: 1) $\forall a, b \in R \rightarrow a \cdot b = a \in R$

\therefore \cdot is a closed

2) $\forall a, b, c \in R \rightarrow a \cdot (b \cdot c) = (a \cdot b) \cdot c$

L.S) $a \cdot b = a$

R.S) $a \cdot c = a$

\therefore \cdot is asso.

3) $\forall a, b, c \in R$ then

$$a.(b+c) = (a.b) + (a.c)$$

$$\text{L.S) } a.(b+c) = a$$

$$\text{R.S) } (a.b) + (a.c) =$$

$$a + a = 2a$$

L.S \neq R.S then $.$ is not distributive over $+$.

$\therefore (R, +, .)$ is not a ring

$$2) a.b = 0 \quad (\text{H.W})$$

Q3/ Suppose that $(R, +, \cdot)$ be a ring then $a \cdot 0 = 0 \cdot a = 0 \quad \forall a \in R$.

proof: since $a \cdot 0 = a \cdot (0 + 0)$

$$= a \cdot 0 + a \cdot 0$$

Hence $0 + a \cdot 0 = a \cdot 0 + a \cdot 0$ by cancellation law get the result.

$$a \cdot 0 = 0$$

Similarly we can proof $0 \cdot a = 0$

Q4/ Let R be a ring with identity 1 , has more than one element then $0 \neq 1$.

Proof: since $R \neq \{0\}$ then there exist $a \in R$ such that $a \neq 0$

Let $1=0$

$$\Rightarrow a = a \cdot 1 = a \cdot 0 = 0$$

$\Rightarrow a = 0$ contradiction

Then $0 \neq 1$

Q5/Prove that a ring $(R,+)$ is commutative iff $(a + b)^2 = a^2 + 2ab + b^2$

Proof: Let $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$\Rightarrow a^2 + ab + ba + b^2 = a^2 + ab + ab + b^2 \text{ (by cancellation law)}$$

$$\Rightarrow ab = ba$$

\therefore A ring $(R,+)$ is commutative.

Conversely: Let $(R,+)$ is commutative then $ab=ba$

$$\text{Now } (a + b)^2 = a^2 + ab + ba + b^2$$

$$= a^2 + ab + ab + b^2 \quad (\text{since } ab = ba)$$

$$= a^2 + 2ab + b^2$$