University of Al-Hamdaniya, College of Education
Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 7

Problems

Q1/ Define two binary operations * and $\circ$ on the set Z of integers as follows:

$$a * b = a + b - 1$$

$$a \circ b = a + b - ab$$

prove that the system $(Z, *, \circ)$ is comm. ring with identity?

Q2/ Let (R,+) comm. group. Determine whether (R,+.) forms a ring with multiplication defined as:

$$\forall a, b \in R$$

1) $a.b = a$

sol: 1) ∀ a,b∈R ⟶ a.b=a∈R

∴ .is a closed

2) ∀ a,b,c∈R⟶a.(b.c)=(a.b).c

L.S) a.b=a

R.S) a.c=a

∴ .is asso.

3) $\forall$ a,b,c$\in$R then

a.(b+c)=(a.b)+(a.c)

L.S) a.(b+c)=a

R.S)(a.b)+(a.c)=

a + a=2a

L.S$\neq$R.S then . is not distributive over +.

$\therefore$(R,+.) is not a ring

2)a.b=0 (H.W)

Q3/ Suppose that (R,+.) be a ring then $a.0 = 0.a = 0 \quad \forall a \in R$.

proof: since $a.0 = a.(0 + 0)$

$$= a.0 + a.0$$

Hence $0 + a.0 = a.0 + a.0$ by cancellation law get the result.

$$a.0 = 0$$

Similarly we can proof $0.a = 0$

Q4/ Let R be a ring with identity 1, has more than one element then $0 \neq 1$.

Proof: since $R \neq \{0\}$ then there exist $a \in R$ such that $a \neq 0$

Let 1=0

$\Rightarrow a = a.1 = a.0 = 0$

$\Rightarrow a = 0$ contodiction

Then $0 \neq 1$

Q5/Prove that a ring (R,+.) is commutative iff $(a+b)^2 = a^2 + 2ab + b^2$

Proof: Let $(a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$\Rightarrow a^2 + ab + ba + b^2 = a^2 + ab + ab + b^2$ (by cancellation law)

⇒ab=ba

∴ A ring (R,+.) is commutative.

Conversely: Let (R,+.) is commutative then ab=ba

Now $(a+b)^2 = a^2 + ab + ba + b^2$

$$= a^2 + ab + ab + b^2 \qquad (since\ ab = ba)$$

$$= a^2 + 2ab + b^2$$