University of Al-Hamdaniya, College of Education

Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 8

Problems

Definition: An element a of a ring (R,+.) said to be nilpotent if $a^n=0$ for some $n \in Z^+$

Example1: $(Z_8,+,.)$ is a ring

$2^3=0$, $4^2=0$, $0^0=0$, $6^3=0$

$\therefore$ nilpotent element in Z_8 are {0,2,4,6}

Example2: $(Z_6,+,.)$ is a ring , find the nilpotent in Z_6.

Sol.: nilpotent in Z_6 is only zero.

Q7/ Prove that in an integral domain the zero element is the only nilpotent element.

Proof: Let $a \in R$ such that $a^n=0$ & assume $a \neq 0$ we must prove that $a=0$

Now $a^n = 0 \rightarrow a.a^{n-1} = 0$

Since $a \neq 0 \rightarrow a^{n-1} = 0$

$\rightarrow a.a^{n-2} = 0 \ , a \neq 0$

$\rightarrow a^{n-2} = 0$

$\vdots$

$\rightarrow a^2 = 0 \rightarrow a.a = 0 \rightarrow a = 0$ C!

$\therefore$ In an integral domain the zero element is the only nilpotent elements.

Definition : let (R,+,.) be a ring then the center of a ring (R,+,.) denoted by cent (R) is defined as:

$$\text{cent}(R) = \{c \in R : cx = xc \ \forall x \in R\}$$

Q8/ prove that (cent (R) ,+,.) is a subring of (R ,+,.).

Proof: 1)cent (R) $\neq \emptyset$ since e $\in$ cent (R)

2)let a, b $\in$ cent (R) , that is

$$ax = xa \qquad \forall x \in R$$

$$bx = xb$$

$$(a - b)x = ax - bx$$

$$xa - xb = x(a - b)$$

$$\therefore a - b \in \text{cent}(R)$$

$$
\begin{aligned}
3)\ (ab)x &= a(bx) \\
&= a(xb) \\
&= (ax)b \\
&= (xa)b \\
&= x(ab)
\end{aligned}
$$

$\therefore a, b \in \text{cent}\ (R)$

$\therefore$ cent (R) is a subring of R.