



University of Al-Hamdaniya, College of
Education

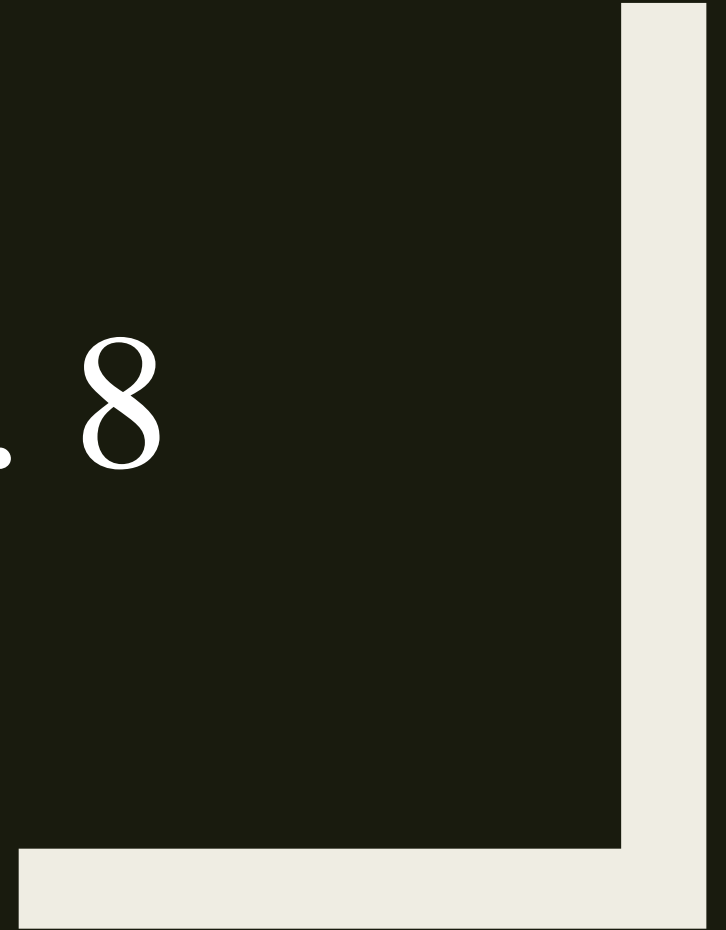
Department of Mathematics

RING THEORY

Level Three

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LECTURE NO. 8



Problems

Definition: An element a of a ring $(R,+)$ said to be nilpotent if $a^n=0$ for some $n \in \mathbb{Z}^+$

Example1: $(\mathbb{Z}_8,+)$ is a ring

$$2^3=0, 4^2=0, 0^0=0, 6^3=0$$

\therefore nilpotent element in \mathbb{Z}_8 are $\{0,2,4,6\}$

Example2: $(\mathbb{Z}_6,+)$ is a ring, find the nilpotent in \mathbb{Z}_6 .

Sol.: nilpotent in \mathbb{Z}_6 is only zero.

Q7/ Prove that in an integral domain the zero element is the only nilpotent element.

Proof: Let $a \in R$ such that $a^n=0$ & assume $a \neq 0$ we must prove that $a=0$

$$\text{Now } a^n = 0 \rightarrow a \cdot a^{n-1} = 0$$

$$\text{Since } a \neq 0 \rightarrow a^{n-1} = 0$$

$$\rightarrow a \cdot a^{n-2} = 0, a \neq 0$$

$$\rightarrow a^{n-2} = 0$$

\vdots

$$\rightarrow a^2 = 0 \rightarrow a \cdot a = 0 \rightarrow a = 0 \text{ C!}$$

\therefore In an integral domain the zero element is the only nilpotent elements.

Definition : let $(R, +, \cdot)$ be a ring then the center of a ring $(R, +, \cdot)$ denoted by $\text{cent}(R)$ is defined as:

$$\text{cent}(R) = \{c \in R : cx = xc \ \forall x \in R\}$$

Q8/ prove that $(\text{cent}(R), +, \cdot)$ is a subring of $(R, +, \cdot)$.

Proof: 1) $\text{cent}(R) \neq \emptyset$ since $e \in \text{cent}(R)$

2) let $a, b \in \text{cent}(R)$, that is

$$ax = xa \quad \forall x \in R$$

$$bx = xb$$

$$(a - b)x = ax - bx$$

$$xa - xb = x(a - b)$$

$$\therefore a - b \in \text{cent}(R)$$

$$\begin{aligned} 3) (ab)x &= a(bx) \\ &= a(xb) \\ &= (ax)b \\ &= (xa)b \\ &= x(ab) \end{aligned}$$

$$\therefore a, b \in \text{cent}(R)$$

$\therefore \text{cent}(R)$ is a subring of R .