الأعداد المعقدة
(COMPLEX NUMBERS)

المحاضرة الأولى

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Definition of Complex Numbers

If a and b are real numbers and i is the imaginary unit, then $a + bi$ is called a ***complex number***.

- a is the ***real part***
- bi is the ***imaginary part***.

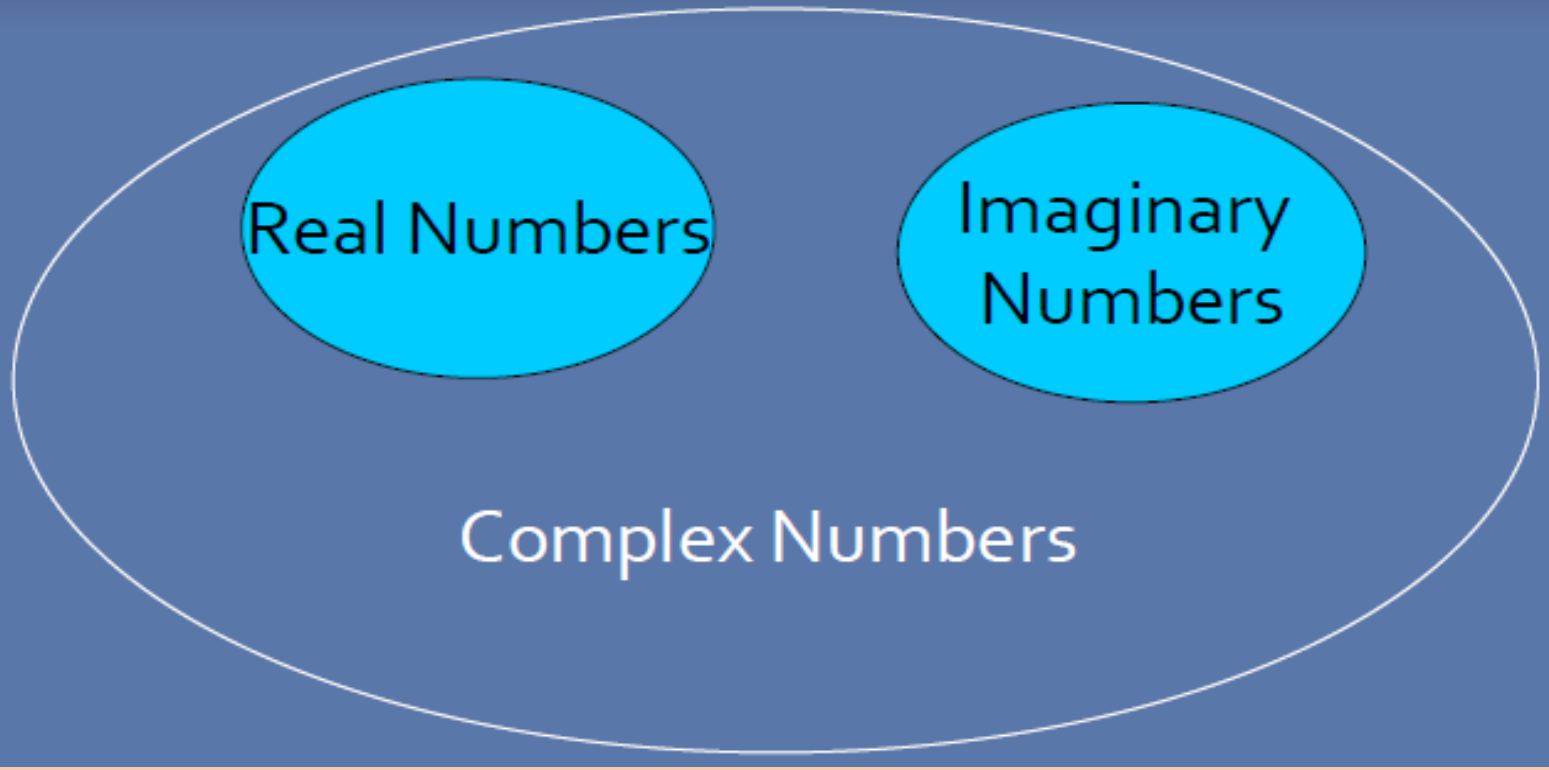
Definition of the Complex Number

$$z = \underbrace{a}_{\text{Real Part}} + \underbrace{bi}_{\text{Imaginary Part}}$$

Rollover for example

COMPLEX NUMBERS

Real numbers and imaginary numbers are subsets of the set of complex numbers.



COMPLEX NUMBERS

Equal complex numbers

Two complex numbers are equal if their real parts are equal and their imaginary parts are equal.

$$\text{If } a + bi = c + di,$$

$$\text{then } a = c \text{ and } b = d$$

Example: $Z = 4 + 3i$

$\mathrm{Re}(Z) = 4 \quad \mathrm{Im}(Z) = 3$

Example

State the real and imaginary parts of $3 + 4i$.

Solution

The real part is 3.

The imaginary part is 4.

Example

State the real and imaginary parts of $-2 + 5i$.

Solution

The real part is -2.

The imaginary part is 5.

IMAGINARY NUMBERS

$i = \sqrt{-1}$

Definition: The number i, called the ***imaginary unit***, is the number such that

$i = \sqrt{-1}$ and $i^2 = -1$

Powers of i

i stands for $\sqrt{-1}$ so: $i^2 = \left(\sqrt{-1}\right)^2 = -1$

$i^4 = (i^2)^2 = (-1)^2 = 1$

For any power of i take out as many i^4's and i^2's as possible and they will all end up as $\pm i$ or ± 1.

Example: $i^{11} = (i^4)^2 i^2 i = 1^2 \times -1 \times i = \underline{\underline{-i}}$

OR: just take out i^2's if you find it easier to remember.

Example: $i^{33} = (i^2)^{16} i = (-1)^{16} i = i$

Summary:

$i = \sqrt{-1}$

$i^2 = -1$ $\quad i^{-2} = -1$

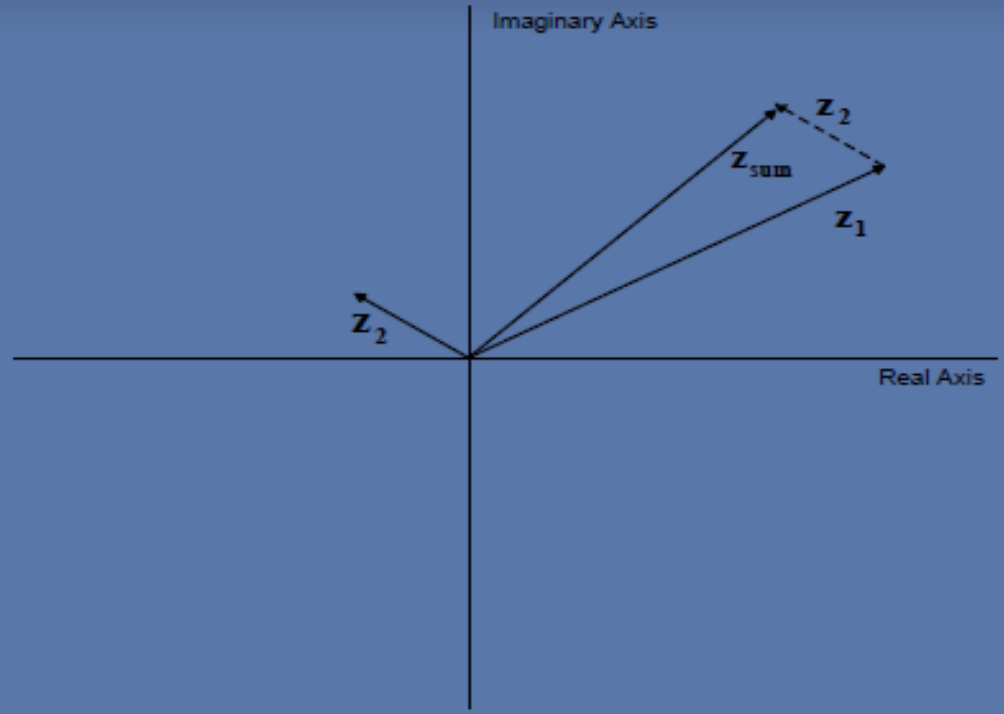
$i^4 = 1$ $\quad i^{-4} = 1$

ADDITION OF COMPLEX NUMBERS

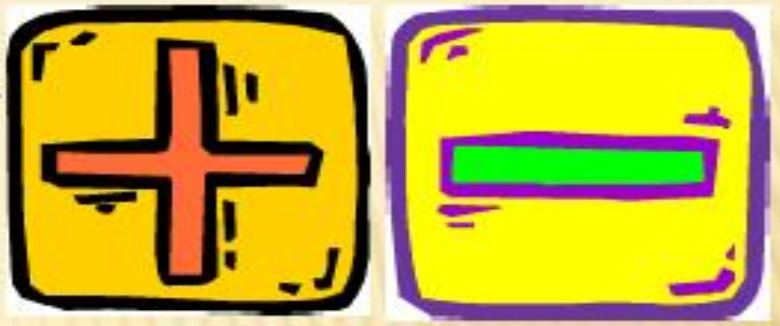
$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

EXAMPLE

$$(2+3i)+(1+5i)$$
$$=(2+1)+(3+5)i$$
$$= 3+8i$$



ADDING OR SUBTRACTING COMPLEX NUMBERS



Let $a + bi$ and $c + di$ be complex numbers.

1. Add/Subtract the Real parts.
2. Add/Subtract the Imaginary parts.

- $(3 + 4i) + (2 - i) = (3 + 2) + (4i - i) = (5 + 3i)$
- $(7 + i) - (3 - i) = (7 - 3) + i(1 - (-1)) = 4 + 2i$

MULTIPLICATION OF COMPLEX NUMBERS

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$$

Example

$$(2+3i)(1+5i)$$
$$=(2-15)+(10+3)i$$
$$=-13+13i$$

MULTIPLYING COMPLEX NUMBERS

Let $a + bi$ and $c + di$ be complex numbers.

1. Multiply the binomials.
2. Convert i^2 to -1 and add the like terms.

$$(3 + 2i)(4 + 5i) = (3 \times 4) + (3 \times (5i)) + ((2i) \times 4) + ((2i) \times (5i))$$
$$= 12 + 15i + 8i + 10i^2$$
$$= 12 + 23i - 10 \text{ (Remenber that } 10i^2 = 10(-1) = -10)$$
$$= 2 + 23i$$

Therefore, $(3 + 2i)(4 + 5i) = 2+23i$

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