الدوال الزائدية

HYPERBOLIC FUNCTIONS

المحاضرة الخامسة عشر

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Hyperbolic Functions.

- The complex hyperbolic cosine and sine are defined by the formulas:

$$\cosh z = \tfrac{1}{2}(e^z + e^{-z}), \qquad \sinh z = \tfrac{1}{2}(e^z - e^{-z}).$$

- Complex Trigonometric and Hyperbolic Functions are related:

$$\cosh iz = \cos z, \qquad \sinh iz = i \sin z.$$

$$\cos iz = \cosh z, \qquad \sin iz = i \sinh z.$$

Since e^z and e^{-z} are entire, it follows from definitions (1) that $\sinh z$ and $\cosh z$ are entire. Furthermore,

$$\frac{d}{dz}\sinh z = \cosh z, \qquad \frac{d}{dz}\cosh z = \sinh z.$$

Because of the way in which the exponential function appears in definitions (1) and in the definitions

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \qquad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

of $\sin z$ and $\cos z$, the hyperbolic sine and cosine functions are closely related to those trigonometric functions:

$$-i\sinh(iz) = \sin z, \qquad \cosh(iz) = \cos z,$$
$$-i\sin(iz) = \sinh z, \qquad \cos(iz) = \cosh z.$$

Some of the most frequently used identities involving hyperbolic sine and cosine functions are

$$\sinh(-z) = -\sinh z, \quad \cosh(-z) = \cosh z,$$

$$\cosh^2 z - \sinh^2 z = 1,$$

$$\sinh(z_1 + z_2) = \sinh z_1 \cosh z_2 + \cosh z_1 \sinh z_2,$$

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$$\sinh z = \sinh x \cos y + i \cosh x \sin y,$$

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$$|\sinh z|^2 = \sinh^2 x + \sin^2 y,$$

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To find the derivatives of sinhz and coshz, we write

$$\frac{d}{dz}\sinh z = \frac{d}{dz}\left(\frac{e^z - e^{-z}}{2}\right) = \frac{1}{2}\frac{d}{dz}(e^z - e^{-z}) = \frac{e^z + e^{-z}}{2} = \cosh z$$

and

$$\frac{d}{dz}\cosh z = \frac{d}{dz}\left(\frac{e^z + e^{-z}}{2}\right) = \frac{1}{2}\frac{d}{dz}(e^z + e^{-z}) = \frac{e^z - e^{-z}}{2} = \sinh z.$$

(a) Observe that

$$\sinh(z+\pi i) = \frac{e^{z+\pi i} - e^{-(z+\pi i)}}{2} = \frac{e^z e^{\pi i} - e^{-z} e^{-\pi i}}{2} = \frac{-e^z + e^{-z}}{2} = -\frac{e^z - e^{-z}}{2} = -\sinh z.$$

(b) Also,

$$\cosh(z+\pi i) = \frac{e^{z+\pi i} + e^{-(z+\pi i)}}{2} = \frac{e^z e^{\pi i} + e^{-z} e^{-\pi i}}{2} = \frac{-e^z - e^{-z}}{2} = -\frac{e^z + e^{-z}}{2} = -\cosh z.$$


Thank You